Entanglement, which-way measurements, and a quantum erasure

Christian Ferrari
Liceo di Locarno, Via F. Chiesa 15, 6600 Locarno, Switzerland

Bernd Braunecker
Department of Physics, University of Basel, Klingelbergstrasse 82, 4056 Basel, Switzerland
(Dated: March 2, 2010. Accepted for American Journal of Physics)

We present a didactical approach to the which-way experiment and the counterintuitive effect of the quantum erasure for one-particle quantum interferences. The fundamental concept of entanglement plays a central role and highlights the complementarity between quantum interference and knowledge of which path is followed by the particle.

I. INTRODUCTION

One-particle quantum interference is one of the most important effects that illustrates the superposition principle and thus the major difference between quantum and classical physics.[1,2] In this paper we propose a simple model based on the Mach-Zehnder interferometer. Our hope is to provide a simple example of quantum superposition and quantum interference.

We consider a modification of the gedanken experiment by Scully, Englert, and Walther,[3] which we reduce to probably the simplest setup that can expose the physics in a concise way. Reference 3 is a highly influential paper and several previous publications discuss and present the experiment in a didactical way.[4–8] The emphasis in these publications ranges from practical realizations of a Mach-Zehnder interferometer to a thorough discussion of the subtleties of quantum physics.

In this paper we show that the fundamental aspects of the experiment can be captured by a minimal model that requires knowledge only of two-level systems and is based on the Mach-Zehnder interferometer. For maximum clarity we avoid an extended discussion of experimental and further theoretical aspects, for which we refer to Refs. 4–8. We also focus on a Mach-Zehnder interferometer with only two detectors at the exit instead of the screen used in Young's two-slit experiment, which corresponds to a continuum of detectors. The Mach-Zehnder interferometer allows us to model the step by step evolution of the state of a quantum particle in the interferometer.

We briefly summarize the experiment proposed in Ref. 3 in which a mechanism is proposed to detect the path ("which-way detection") of a particle passing through a Young interferometer (see Fig. 1). An atom is emitted by a source S, passes through two slits, and is detected on a screen D. Directly after leaving the source, the atom is brought into an excited state by a laser. Two cavities C_1 and C_2 are placed in front of the slits of the Young interferometer. When passing through the cavities the atom emits a photon and relaxes to its ground state. To know which path was taken by the atom it is sufficient to see whether the photon is in C_1 or C_2. Important for this experiment is that the trajectory of the atom through the slits remains otherwise unperturbed.

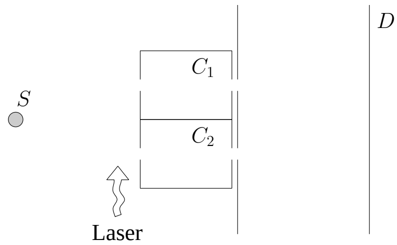

FIG. 1: Schematic of the experiment proposed in Ref. 3.

Due to the emission of the photon in cavity C_1 or C_2 the usual interference pattern at the screen S is destroyed. The interference disappears even without explicit detection of the photon. It is sufficient to transfer the potential which-way information to the photon state. However, by allowing the photon emitted in C_1 or C_2 to be reabsorbed by an auxiliary atom, a *quantum eraser*, the information of the atom's path can be erased, and the interference pattern at the screen can be restored. This result was confirmed experimentally by Dürr, Nonn, and Rempe[9] using a modified Mach-Zehnder interferometer (see also Refs. 10 and 11).

II. THE MACH-ZEHNDER INTERFEROMETER

We consider the Mach-Zehnder interferometer shown in Fig. 2. It consists of a source S, which emits particles into the interferometer along the x direction such that at any given time, a maximum of a single particle is in the interferometer. (See Ref. 2 for a discussion of the first experiment realizing single-particle interference.) The particles first hit beam splitter BS_1 through which they are transmitted to path A or deflected to path B. Two mirrors M_A and M_B let these paths cross again at a second beam splitter BS_2. At the exit of BS_2 are two detectors, D_X along the x direction and D_Y along the y direction, as indicated in Fig. 2.

An explanation of the Mach-Zehnder interferometer is given in Ref. 12. For completeness and to establish the notation we summarize the main physics of the Mach-Zehnder interferometer. The state inside the interferometer can be modeled as a two-level system, for instance,

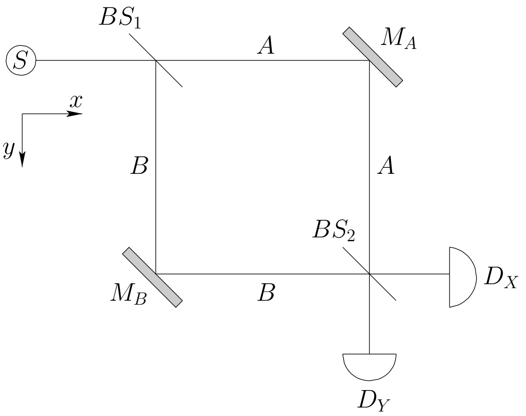

FIG. 2: The Mach-Zehnder interferometer. A particle (atom) is emitted by the source S and travels to the detectors D_X and D_Y. At the beam splitter BS_1 it is deflected to path A or path B. With the mirrors M_A and M_B the particle interferes with itself at the beam splitter BS_2 before entering the detectors. A and B have the same length, and therefore quantum interference leads to detection probability equal to 1 in D_X and 0 in D_Y, in contrast to the classical expectation of equal probabilities of 1/2 for both detectors.

by associating the states $|x\rangle$ and $|y\rangle$ with the particle at any time corresponding to its direction of propagation inside the interferometer (see Fig. 2).

If both paths A and B have the same length, we can neglect the phase of the particle on the trajectories between the beam splitters and mirrors. The effect of the interferometer on the particle state is then given by the sequence of unitary transformations $BS_1 \to M_{A,B} \to BS_2$. A symmetric and equilibrated beam splitter is described by the unitary transformation[13]

$$\text{Beam splitter:} \begin{cases} |x\rangle \longrightarrow \frac{1}{\sqrt{2}}(|x\rangle + i|y\rangle), \\ |y\rangle \longrightarrow \frac{1}{\sqrt{2}}(|y\rangle + i|x\rangle), \end{cases} \tag{1}$$

while the combination of the mirrors M_A and M_B acts as $|x\rangle \to i|y\rangle$ and $|y\rangle \to i|x\rangle$. We combine these transformation and see that the interferometer acts as

$$\text{Mach-Zehnder interferometer:} \begin{cases} |x\rangle \longrightarrow e^{i\pi}|x\rangle, \\ |y\rangle \longrightarrow e^{i\pi}|y\rangle. \end{cases} \tag{2}$$

At the exit of the Mach-Zehnder interferometer the detector D_X measures the component $|x\rangle$ of the outgoing state $|\psi_{\text{out}}\rangle$, and D_Y, the component $|y\rangle$ with the probabilities

$$\text{Prob}\{X\} = \|P_{|x\rangle}|\psi_{\text{out}}\rangle\|^2 = |\langle x|\psi_{\text{out}}\rangle|^2, \tag{3a}$$
$$\text{Prob}\{Y\} = \|P_{|y\rangle}|\psi_{\text{out}}\rangle\|^2 = |\langle y|\psi_{\text{out}}\rangle|^2, \tag{3b}$$

where $P_{|x\rangle}$ and $P_{|y\rangle}$ are the projectors onto $|x\rangle$ and $|y\rangle$, respectively. Particles are injected from the source along the x direction, that is, in state $|x\rangle$. From Eq. (2) it therefore follows that the probability of measuring the particle in detector D_X is $\text{Prob}\{X\} = 1$, and the probability of measuring it in detector D_Y is $\text{Prob}\{Y\} = 0$. This result is in contrast to the expected classical result, which is $\text{Prob}\{X\} = \text{Prob}\{Y\} = 1/2$. This effect is known as *one-particle quantum interference* and is one of the typical non-classical and counterintuitive effects of the quantum physics.[14] Increasing the length of one of the paths A or B leads to an additional phase difference of the states before BS_2 and can be used to control of the interference and hence the probabilities $\text{Prob}\{X\}$ and $\text{Prob}\{Y\}$.[12]

III. WHICH-WAY DETECTOR

The quantum interference effect is destroyed if we put additional (nondestructive) detectors $\tilde{D}_A$ on path A and $\tilde{D}_B$ on path B (see Fig. 3) to detect which path was chosen by the particle. A detection by $\tilde{D}_A$ projects the particle onto the state $|x\rangle$ and tells us that path A was taken. A detection by $\tilde{D}_B$ projects onto $|y\rangle$ and tells us that path B was taken. Hence the state at the exit of the interferometer is fully determined by the action of the beam splitter BS_2 on the incoming state from either path A or path B. The final beam splitting leads to the probabilities $\text{Prob}\{X\} = \frac{1}{2}$ and $\text{Prob}\{Y\} = \frac{1}{2}$, equivalent to the classical result and independent of the result detected by $\tilde{D}_X$ and $\tilde{D}_Y$. The quantum interference disappears, showing that the concepts of “quantum interference” and “knowledge of the path” are complementary.

Note that the interference disappears as soon as the information on the path is stored in the system state. It is not a “uncontrolled” perturbation of the state of the quantum particle that destroys the interference.

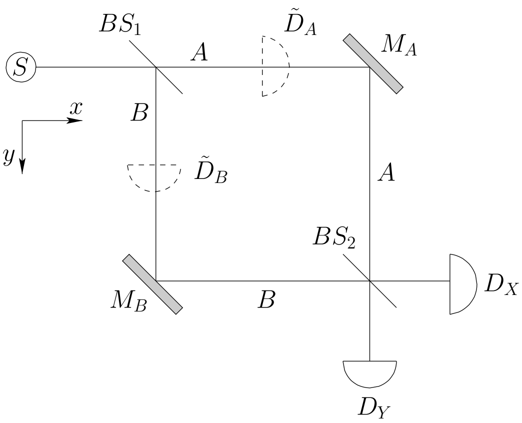

FIG. 3: Which-way detection in the Mach-Zehnder interferometer. Additional (nondestructive) detectors $\tilde{D}_A$ and $\tilde{D}_B$ are placed on the paths A and B. The entanglement of the particle with $\tilde{D}_A$ and $\tilde{D}_B$ provides information on which path was taken by it. This information destroys the quantum interference and results in detection probabilities equal to 1/2 in D_X and D_Y corresponding to the classical result.

IV. WHICH-WAY ENTANGLER

A remarkable property of quantum physics is that the interference vanishes by the mere presence of the detectors $\tilde{D}_A$ and $\tilde{D}_B$, even if the result of the measurement is not classically read out (which would correspond to a projection on the path taken A or B). This property can be illustrated by a simple extension of the Mach-Zehnder interferometer, which also lets us show that a detection takes place by *entanglement* between the particle and the detector. We call this model a *which-way entangler*. We start from the which-way detector shown in Fig. 3. In addition, we now assume that the particle is emitted into the interferometer in an excited state $|e\rangle$. The detection by detectors $\tilde{D}_A$ or $\tilde{D}_B$ relaxes the particle into its ground state $|g\rangle$ by emission of a photon. We denote the photon state by $|A\rangle$ or $|B\rangle$, determined by which detector received the photon, and use the notation $|0\rangle$ for the absence of any photon.

The state at the entrance of the interferometer is therefore

$$|\Psi_{\rm in}\rangle = |x\rangle \otimes |0\rangle \otimes |e\rangle. \quad (4)$$

After the first beam splitter the state is

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}\Big[|x\rangle + i|y\rangle\Big] \otimes |0\rangle \otimes |e\rangle. \quad (5)$$

The action of the detectors $\tilde{D}_A$ or $\tilde{D}_B$ leads to the entangled state

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}\Big[|x\rangle \otimes |A\rangle + i|y\rangle \otimes |B\rangle\Big] \otimes |g\rangle. \quad (6)$$

Note that we do not classically read out the detectors here but keep the quantum coherent superposition between the path A and path B detections by transferring the which-way information into the photon state. As long as the photon state is not measured, the superposition is maintained. This state becomes after the mirrors

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}}\Big[i|y\rangle \otimes |A\rangle - |x\rangle \otimes |B\rangle\Big] \otimes |g\rangle, \quad (7)$$

and as the final state after the second beam splitter

$$|\Psi_{\rm out}\rangle = \frac{1}{2}\Big[\big(i|y\rangle - |x\rangle\big) \otimes |A\rangle - \big(|x\rangle + i|y\rangle\big) \otimes |B\rangle\Big] \otimes |g\rangle \quad (8a)$$

$$= \frac{1}{2}\Big[i|y\rangle \otimes \big(|A\rangle - |B\rangle\big) - |x\rangle \otimes \big(|A\rangle + |B\rangle\big)\Big] \otimes |g\rangle. \quad (8b)$$

We see that even though we keep the superposition of the which-way results, the interference effect at the final detectors D_X and D_Y is destroyed, and the detection probabilities correspond to the classical results

$${\rm Prob}\{X\} = \|P_{|x\rangle} \otimes I \otimes I|\Psi_{\rm out}\rangle\|^2 = \frac{1}{4}\||A\rangle + |B\rangle\|^2 = \frac{1}{2}, \quad (9a)$$

$${\rm Prob}\{Y\} = \|P_{|y\rangle} \otimes I \otimes I|\Psi_{\rm out}\rangle\|^2 = \frac{1}{4}\||A\rangle - |B\rangle\|^2 = \frac{1}{2}. \quad (9b)$$

However, in addition to the which-way detection alone, we have now transmitted the which-way information into the photon states $|A\rangle$ and $|B\rangle$. Once the photon is emitted, it becomes *entangled* with the quantum particle state, which means that we can no longer write the state as a simple product $|\Psi\rangle = |\text{particle}\rangle \otimes |\text{photon}\rangle$, as is clearly seen with $|\Psi_2\rangle$ in Eq. (6).

V. QUANTUM ERASER

We now show that the which-way information can be erased in a simple way, which restores the quantum interference at the output of the Mach-Zehnder interferometer.

Consider the Mach-Zehnder interferometer, modified in the following way. We assume that the photon after emission in one of the two $\tilde{D}$ detectors travels to an auxiliary atom $\mathcal{E}$, the *quantum eraser*, where it can be absorbed (see Fig. 4). To activate the quantum eraser, the observer has to open a channel c connecting detectors $\tilde{D}_A$ and $\tilde{D}_B$ to atom $\mathcal{E}$. This opening can be done at any time after the emission of the photons, even when the quantum particle has left the Mach-Zehnder interferometer.[4,5]

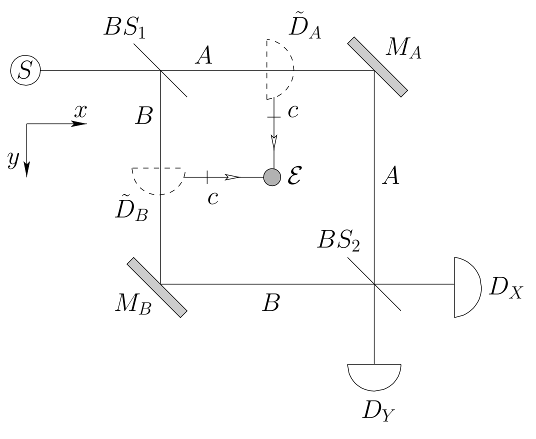

FIG. 4: Quantum eraser for the Mach-Zehnder interferometer. The which-way detection takes place by emission of a photon in $\tilde{D}_A$ or $\tilde{D}_B$. If channel c is open, the photon can be absorbed, with some probability, by an auxiliary atom $\mathcal{E}$, the quantum eraser. The absorption erases the entanglement with the detectors and restores the quantum interference at the exit of the interferometer.

Let $|\gamma\rangle$ be the initial (ground) state of the quantum eraser $\mathcal{E}$ and let $|\varepsilon\rangle$ be its excited state. We consider the evolution of the system "atom + photon + quantum eraser." If the channel c is closed, then the state just before the final detection is identical to Eq. (8) with the additional state $|\gamma\rangle$ of $\mathcal{E}$,

$$|\Phi_{\text{out}}\rangle = |\Psi_{\text{out}}\rangle \otimes |\gamma\rangle \tag{10a}$$

$$= \frac{1}{2}\Big[i|y\rangle \otimes (|A\rangle - |B\rangle) - |x\rangle \otimes (|A\rangle + |B\rangle)\Big] \otimes |g\rangle \otimes |\gamma\rangle, \tag{10b}$$

and we obtain the same probabilities 1/2 at the final detectors as in Sec. IV.

In contrast, if channel c is open, the photon travels from the detectors $\tilde{D}_A$ and $\tilde{D}_B$ to the eraser $\mathcal{E}$ where with some probability it can be absorbed by exciting the quantum eraser. We stress that the absorption of the photon is probabilistic and depends on the precise superposition of the $|A\rangle$ and $|B\rangle$ components of the photon state at $\mathcal{E}$ and on the cross-section of the absorption process. An absorption (erasure) that occurs with certainty would be a nonunitary transformation which is forbidden by quantum physics and would allow for paradoxes such as superluminous transmission of information, that is, a violation of the no-signalling-theorem.[15,16]

Therefore the quantum erasure occurs only for some outcomes of the interference experiments. For those cases where the photon is absorbed by $\mathcal{E}$, the system state is projected onto

$$|\Phi\rangle = -|x\rangle \otimes |0\rangle \otimes |g\rangle \otimes |\varepsilon\rangle. \tag{11}$$

This state is identical (for the injected particle) to the usual action of the Mach-Zehnder interferometer expressed by Eq. (2), and therefore the single-particle interference at the detectors D_X and D_Y is restored. However, the probabilities $\text{Prob}\{X\}$ and $\text{Prob}\{Y\}$ are now replaced by the conditional probabilities $\text{Prob}\{X|\text{abs}\}$ and $\text{Prob}\{Y|\text{abs}\}$, which involve the preselection of the measurements to only those cases where the photon has actually been absorbed by the quantum eraser.[15] As a result we obtain

$$\text{Prob}\{X|\text{abs}\} = \|(P_{|x\rangle} \otimes I \otimes I \otimes I)|\Phi\rangle\|^2 = 1, \tag{12a}$$

$$\text{Prob}\{Y|\text{abs}\} = \|(P_{|y\rangle} \otimes I \otimes I \otimes I)|\Phi\rangle\|^2 = 0. \tag{12b}$$

We see that the erasure of the which-path information by the absorption of the photon by the quantum eraser completely restores the original quantum interference.

VI. CONCLUSION

We have presented a simple model that requires knowledge only of two-level systems. Nonetheless, it allows us to explain interesting effects about one-particle quantum interference: Quantum interference appears when a particle can take different indistinguishable paths to arrive at a detector. The knowledge of which path was taken is obtained by entanglement between the quantum particle and a detector on the path. The loss of the one-particle quantum interference is an illustration that this entanglement changes the state of the particle. The interference can be restored by using the quantum eraser, which disentangles the particle and detector states, and thus also erases any which-way information. Note that a noisy environment acts in a similar way as the which-path detector and destroys the quantum interference by getting entangled through interaction with the particle. However, this effect is generally uncontrolled and not reversible by a quantum eraser, and the result is a purely classically operating Mach-Zehnder interferometer.

The complementarity between "quantum interference" and "knowledge of the path" in this simple model is manifestly evident: "quantum interference" corresponds to a factorized (product) state, "knowledge of the path" to an entangled state.

Acknowledgments

The authors thank V. Scarani for stimulating and helpful discussions, M. Bahrami for useful correspondence, and the anonymous referees of Am. J. Phys. for a careful review and constructive comments. B.B. acknowledges the support by the Swiss NCCR Nanoscience (Basel) and SNF.

1 R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1963).
2 H. Rauch, A. Zeilinger, G. Badurek, A. Wilfing, W. Bauspiess, and U. Bonse, "Verification of coherent spinor rotations of fermions," Phys. Lett. A **54**, 425–427 (1975).
3 M. O. Scully, B.-G. Englert, and H. Walther, "Quantum optical tests of complementarity," Nature **351**, 111–116 (1991).
4 B.-G. Englert, M. O. Scully, and H. Walther, "Quantum erasure in double-slit interferometers with which-way detectors," Am. J. Phys. **67**, 325–329 (1999).
5 U. Mohrhoff, "Objectivity, retrocausation, and the experiment of Englert, Scully, and Walther," Am. J. Phys. **67**, 330–335 (1999).
6 M. B. Schneider and I. A. LaPuma, "A simple experiment for discussion of quantum interference and which-way measurement," Am. J. Phys. **70**, 266–271 (2002).
7 T. L. Dimitrova and A. Weis, "The wave-particle duality of light: A demonstration experiment," Am. J. Phys. **76**, 137–142 (2008).
8 E. J. Galvez, C. H. Holbrow, M. J. Pysher, J. W. Mar-

tin, N. Courtemanche, L. Heilig, and J. Spencer, "Interference with correlated photons: Five quantum mechanics experiments for undergraduates," Am. J. Phys. **73**, 127–140 (2005).

[9] S. Dürr, T. Nonn, and G. Rempe, "Origin of quantum-mechanical complementarity probed by a 'which-way' experiment in an atom interferometer," Nature **395**, 33–37 (1998).

[10] M. G. Raymer and S. B. Yang, "Information and complementarity in a proposed which-path experiment using photons," J. Mod. Opt. **39**, 1221–1231 (1992).

[11] A. Gogo, W. D. Snyder, and M. Beck, "Comparing quantum and classical correlations in a quantum eraser," Phys. Rev. A **71**, 052103-1–6 (2005).

[12] V. Scarani and A. Suarez, "Introducing quantum mechanics: One-particle interferences," Am. J. Phys. **66**, 718–721 (1998).

[13] A. Zeilinger, "General properties of lossless beam splitters in interferometry," Am. J. Phys. **49**, 882–883 (1981).

[14] V. Scarani, *Quantum Physics: A First Encounter* (Oxford University Press, Oxford, 2006).

[15] G. C. Ghirardi, A. Rimini, and T. Weber, "A general argument against superluminal transmission through the quantum mechanical measurement process," Lett. Nuovo Cimento **27**, 293–298 (1980).

[16] M. Bahrami and A. Shafiee, "Postponing the past: An operational analysis of delayed-choice experiments," Found. Phys. **40**, 55–92 (2010).